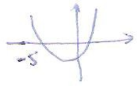
Ex 241

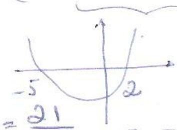
a) • $\lim\limits_{x\to 2} \dfrac{x^2-4}{x^2+3x-10} \overset{\frac{0}{0}}{=} \lim\limits_{x\to 2} \dfrac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x+5)} = \lim\limits_{x\to 2} \dfrac{x+2}{x+5} = \dfrac{2+2}{2+5} = \dfrac{4}{7}$

• $\lim\limits_{x\to -5^+} \dfrac{x^2-4}{\underbrace{x^2+3x-10}} = \dfrac{25-4}{0^-} = \dfrac{21}{0^-} = -\infty$

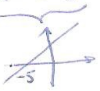


• $\lim\limits_{x\to +\infty} \dfrac{x^2-4}{x^2+3x-10} \overset{\left(\frac{\infty}{\infty}\right)}{=} \lim\limits_{x\to +\infty} \dfrac{x^2}{x^2} = 1$

b) • $\lim\limits_{x\to 2^+} \dfrac{x^2-4}{(x-2)^2(x+5)} \overset{\frac{0}{0}}{=} \lim\limits_{x\to 2^+} \dfrac{\cancel{(x-2)}(x+2)}{(x-2)^{\cancel{2}}\underbrace{(x+5)}} = \dfrac{4}{0^+} = +\infty$



• $\lim\limits_{x\to -5^-} \dfrac{x^2-4}{\underbrace{(x-2)^2}_{>0}\underbrace{(x+5)}} = \dfrac{25-4}{(-7)^2\times 0} = \dfrac{21}{0^-} = -\infty$



• $\lim\limits_{x\to -\infty} \dfrac{x^2-4}{(x-2)^2(x+5)} \overset{\left(\frac{\infty}{\infty}\right)}{=} \lim\limits_{x\to -\infty} \dfrac{x^2-4}{(x^2-4x+4)(x+5)} = \lim\limits_{x\to -\infty} \dfrac{x^2-4}{x^3+x^2-16x+20}$

$= \lim\limits_{x\to -\infty} \dfrac{\cancel{x^2}}{x^{\cancel{3}}} = \lim\limits_{x\to -\infty} \dfrac{1}{x} = \dfrac{1}{-\infty} = 0$

c) • $\lim\limits_{x\to -1} \dfrac{x^2-x-2}{2x^2-x-3} \overset{\left(\frac{0}{0}\right)}{=} \lim\limits_{x\to -1} \dfrac{\cancel{(x+1)}(x-2)}{2(x-\frac{3}{2})\cancel{(x+1)}} = \lim\limits_{x\to -1} \dfrac{x-2}{2(x-\frac{3}{2})} = \dfrac{-1-2}{2(-1-\frac{3}{2})}$

$= \dfrac{-3}{\cancel{2}\left(-\frac{5}{\cancel{2}}\right)} = \dfrac{3}{5}$

• $\lim\limits_{x\to -\infty} \dfrac{x^2-x-2}{2x^2-x-3} \overset{\left(\frac{\infty}{\infty}\right)}{=} \lim\limits_{x\to -\infty} \dfrac{\cancel{x^2}}{2\cancel{x^2}} = \dfrac{1}{2}$

d) $\lim\limits_{x\to -2} \dfrac{x^2+2x}{x+2} \overset{\left(\frac{0}{0}\right)}{=} \lim\limits_{x\to -2} \dfrac{x\cancel{(x+2)}}{\cancel{x+2}} = \lim\limits_{x\to -2} x = -2$

e) $\lim\limits_{x\to -1} \dfrac{x^2+2x+1}{x+1} \overset{\left(\frac{0}{0}\right)}{=} \lim\limits_{x\to -1} \dfrac{(x+1)^{\cancel{2}}}{\cancel{(x+1)}} = \lim\limits_{x\to -1} (x+1) = -1+1 = 0$

f) $\lim\limits_{x\to 3} \frac{x-3}{x^2-9} \overset{\frac{0}{0}}{=} \lim\limits_{x\to 3}\frac{(\cancel{x-3})}{(\cancel{x-3})(x+3)} = \lim\limits_{x\to 3}\frac{1}{x+3} = \frac{1}{3+3} = \frac{1}{6}$

g) $\lim\limits_{h\to 0}\frac{(2+h)^2-2^2}{h} \overset{\frac{0}{0}}{=} \lim\limits_{h\to 0}\frac{\cancel{4}+4h+h^2-\cancel{4}}{h} = \lim\limits_{h\to 0}\frac{4h+h^2}{h} =$

$= \lim\limits_{h\to 0}(4+h) = 4+0 = 4$

h) $\lim\limits_{x\to 1}\frac{\frac{1}{x}-1}{x-1} \overset{\frac{0}{0}}{=} \lim\limits_{x\to 1}\frac{\frac{1-x}{x}}{x-1} = \lim\limits_{x\to 1}\frac{1-x}{x(x-1)} = \lim\limits_{x\to 1}\frac{-\cancel{(x-1)}}{x\cancel{(x-1)}}$

$= \lim\limits_{x\to 1} -\frac{1}{x} = -\frac{1}{1} = -1.$

i) $\lim\limits_{x\to -2}\frac{\frac{x}{2}+1}{2+x} \overset{\frac{0}{0}}{=} \lim\limits_{x\to -2}\frac{\frac{x+2}{2}}{2+x} = \lim\limits_{x\to -2}\frac{\cancel{x+2}}{2(\cancel{2+x})} =$

$= \lim\limits_{x\to -2}\frac{1}{2} = \frac{1}{2}.$

j) • $\lim\limits_{x\to -\infty}\frac{x^2-1}{|x-1|\cdot x} = \lim\limits_{x\to -\infty}\frac{x^2-1}{(-x+1)\cdot x}$ $\quad |x-1| = \begin{cases} x-1, & x\geq 1\\ -x+1, & x<1\end{cases}$

$= \lim\limits_{x\to -\infty}\frac{x^2-1}{-x^2+x} \overset{\frac{\infty}{\infty}}{=} \lim\limits_{x\to -\infty}\frac{\cancel{x^2}}{-\cancel{x^2}} = -1$

• $\lim\limits_{x\to 1}\frac{x^2-1}{|x-1|\cdot x}$

$\nexists$

Não existe.

• $\lim\limits_{x\to 1^+}\frac{x^2-1}{(x-1)\cdot x} \overset{\left(\frac{0}{0}\right)}{=} \lim\limits_{x\to 1^+}\frac{(\cancel{x-1})(x+1)}{(\cancel{x-1})\cdot x}$

$= \lim\limits_{x\to 1^+}\frac{x+1}{x} = 2$

• $\lim\limits_{x\to 1^-}\frac{x^2-1}{(-x+1)x} \overset{\left(\frac{0}{0}\right)}{=} \lim\limits_{x\to 1^-}\frac{(\cancel{x-1})(x+1)}{-(\cancel{x-1})x}$

$= \lim\limits_{x\to 1^-}\frac{x+1}{-x} = \frac{1+1}{-1} = \frac{2}{-1} = -2$