Ex 248

a) $\lim_{x\to 0} \frac{e^{3x}-1}{2x} = \frac{1}{2} \underbrace{\lim_{x\to 0} \frac{e^{3x}-1}{3x}}_{L.N.} \cdot 3 = \frac{1}{2} \times 1 \times 3 = \frac{3}{2}$

b) $\lim_{x\to 0} \frac{x}{1-e^{0,1x}} = - \underbrace{\lim_{x\to 0} \frac{0,1x}{e^{0,1x}-1}}_{L.N.} \cdot \frac{1}{0,1} = - 1 \times \frac{1}{0,1} = -10$

c) $\lim_{x\to 0} \frac{0,5x}{\ln(x+1)^2} = \lim_{x\to 0} \frac{0,5\,x}{2\ln(x+1)} = \frac{0,5}{2} \underbrace{\lim_{x\to 0} \frac{x}{\ln(x+1)}}_{L.N.}$

$= \frac{1}{4} \times 1 = \frac{1}{4}$

d) $\lim_{x\to 2} \frac{e^{x}-e^{2}}{x-2} = \lim_{x\to 2} \frac{e^2(e^{x-2}-1)}{x-2} =$

$= e^2 \underbrace{\lim_{x\to 2} \frac{e^{x-2}-1}{x-2}}$ $= e^2 \times 1 = e^2$

$x\to 2$

$x-2\to 0 \quad y = x-2 \quad \lim_{y\to 0} \frac{e^{y}-1}{y} = 1$

e) $\lim_{x\to -1} \frac{e^{x}-e^{-1}}{x^2+x} = \lim_{x\to -1} \frac{e^{-1}(e^{x+1}-1)}{x(x+1)} =$

$= \lim_{x\to -1} \frac{e^{-1}}{x} \cdot \underbrace{\lim_{x\to -1} \frac{e^{x+1}-1}{x+1}}$ $= \frac{e^{-1}}{-1} \cdot 1 = -\frac{1}{e}$

$x+1\to 0$

$y = x+1 \quad \lim_{y\to 0} \frac{e^{y}-1}{y} = 1$

f) $\lim_{x\to 2} \frac{\ln(x-1)}{x^2-4} = \lim_{x\to 2} \frac{\ln(x-1)}{(x-2)(x+2)} = \underbrace{\lim_{x\to 2} \frac{\ln(x-1)}{x-2}}_{*} \cdot \lim_{x\to 2} \frac{1}{x+2}$

$x\to 2 \qquad y = x-2 \Leftrightarrow x = y+2$

$x-2\to 0$

$* \quad \lim_{y\to 0} \frac{\ln(y+1)}{y} = 1 \qquad = 1 \times \frac{1}{2+2} = \frac{1}{4}$

g) $\lim_{x\to 0} \frac{e^{x}-x-1}{x} = \lim_{x\to 0} \left(\frac{e^{x}-1}{x} - \frac{x}{x}\right) = \underbrace{\lim_{x\to 0} \frac{e^{x}-1}{x}}_{L.N.} - \lim_{x\to 0} 1$

$= 1 - 1 = 0$