Ex 249

a) $\lim_{x\to e} \frac{\ln x - 1}{x-e} = \lim_{y\to 0} \frac{\ln(y+e)-1}{y} = \lim_{y\to 0} \frac{\ln(y+e)-\ln e}{y} =$

$x-e\to 0$

$y = x-e$

$= \lim_{y\to 0} \frac{\ln\left(\frac{y+e}{e}\right)}{y} = \lim_{y\to 0} \frac{\ln\left(\frac{y}{e}+1\right)}{y} =$

$= \underbrace{\lim_{y\to 0} \frac{\ln\left(\frac{y}{e}+1\right)}{\frac{y}{e}}}_{L.N.} \cdot \frac{1}{e} = 1 \times \frac{1}{e} = \frac{1}{e}$

b) $\lim_{x\to 0} \frac{1-3^x}{2x} = \lim_{x\to 0} \frac{1-e^{\ln 3^x}}{2x} = \lim_{x\to 0} \frac{1-e^{x\ln 3}}{2x} =$

$= \lim_{x\to 0} \frac{-(e^{x\ln 3}-1)}{2x} = -\frac{1}{2} \underbrace{\lim_{x\to 0} \frac{e^{x\ln 3}-1}{x\ln 3}}_{L.N} \cdot \ln 3 =$

$= -\frac{1}{2} \times 1 \times \ln 3 = \frac{-\ln 3}{2}$

c) $\lim_{x\to+\infty} \frac{2^x(x+1)}{1-3^x} \overset{\left(\frac{\infty}{\infty}\right)}{=} \lim_{x\to+\infty} \frac{\frac{2^x(x+1)}{3^x}}{\frac{1}{3^x}-\frac{3^x}{3^x}} = \frac{\lim_{x\to+\infty} \frac{2^x(x+1)}{3^x}}{\lim_{x\to+\infty}\left(\frac{1}{3^x}-1\right)}$

$\frac{1}{+\infty} - 1 = 0-1 = -1$

$= -\lim_{x\to+\infty} \frac{2^x(x+1)}{3^x} = -\lim_{x\to+\infty} \frac{1}{\frac{3^x}{2^x(x+1)}} =$

$= -\lim_{x\to+\infty} \frac{1}{\frac{\left(\frac{3}{2}\right)^x}{x+1}} = \frac{-1}{\lim_{x\to+\infty} \frac{(3/2)^x}{x+1}} \text{ L.N.} = \frac{-1}{+\infty} = 0$

d) $\lim_{x\to+\infty} \frac{3x-\ln x}{5x^2} = \lim_{x\to+\infty} \frac{3x}{5x^2} - \lim_{x\to+\infty} \frac{\ln x}{5x^2} =$

$= \lim_{x\to+\infty} \frac{3}{5x} - \underbrace{\lim_{x\to+\infty} \frac{\ln x}{x}}_{L.N.} \cdot \lim_{x\to+\infty} \frac{1}{5x} =$

$= \frac{3}{5(+\infty)} - 0 \times \frac{1}{5(+\infty)} = 0 - 0 = 0$